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Mathematical Modeling for Evaluating the Hydraulic and Thermodynamic Performance of the Hamada–Zawiya Pipeline: A Novel Net Exergy Improvement through Heating (NEI_H) Index and Enhanced Light-Crude-Oil Transport Efficiency

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Abstract:
Long-distance crude-oil transportation through pipelines is subject to significant operational and hydraulic challenges arising from thermal variations, which directly affect the fluid's rheological and physical properties. This study develops an integrated mathematical modeling framework to evaluate the influence of temperature on the rheological, hydraulic, and thermodynamic behavior of Hamada light crude oil (42.6° API) flowing through the 387-km Hamada–Zawiya pipeline. The Andrade model was employed to represent the temperature dependence of dynamic viscosity, while the Swamee–Jain equation was used to calculate the friction factor under turbulent-flow conditions. The analysis further introduces the Net Exergy Improvement through Heating (NEI_H) index, which quantitatively balances the mechanical-exergy savings obtained from reducing viscous losses against the thermal exergy consumed in heating the crude oil.
Simulation results showed that increasing the operating temperature sharply reduced the dynamic viscosity from 11.84 cP at 10°C to 1.85 cP at 100°C, accompanied by an increase in the Reynolds number (Re) from 42,422 to 250,373. The total pressure drop decreased from 137.02 bar to 91.41 bar as flow resistance declined, thereby generating substantial mechanical-energy savings. The NEIH index also demonstrated a clear advantage over the conventional index: it increased from a negative value of −5.3437 at 10°C to a maximum of 6.4777 at 20°C, revealing an improved balance between thermal and mechanical exergy and enabling the identification of the most energy-efficient thermal operating range. The proposed model provides a quantitative framework for evaluating crude-oil pipeline performance and supporting decisions aimed at optimizing energy consumption and thermodynamic efficiency.

Keywords: Hamada crude oil; Pipeline transportation; Hydraulic performance; Exergy analysis; Net Exergy Improvement through Heating (NEI_H); Andrade model; Pressure drop.

1-Introduction:
Pipelines constitute the most efficient means of transporting crude oil over long distances. Nevertheless, their operational performance is strongly affected by ambient thermal variations, which directly alter the oil's physical and rheological properties, particularly viscosity and density. These changes modify the flow characteristics—including the Reynolds number, friction factor, and pressure drop—and consequently affect the required pumping power, energy consumption, and overall operating efficiency.
Most previous studies have focused either on the hydraulic analysis of frictional losses or on characterizing the temperature dependence of viscosity. Only limited attention has been given to linking these variables to an exergy-based thermodynamic assessment, which makes it possible to evaluate energy quality and identify irreversibilities caused by friction and heat transfer. Moreover, assessments of crude-oil heating commonly emphasize viscosity reduction and improved flowability without quantitatively balancing pumping-energy savings against the thermal energy consumed [1,2].

The research gap lies in the absence of a unified modeling framework that integrates the thermo-rheological behavior of crude oil with the hydraulic response of the pipeline while simultaneously incorporating an exergy analysis capable of evaluating the thermodynamic feasibility of heating. This limitation constrains the determination of optimal operating conditions from a perspective that combines hydraulic performance with efficient energy utilization.

To address this gap, the present study develops a mathematical model that combines the Andrade viscosity–temperature relationship with the Swamee–Jain equation for friction-factor calculation. The model is applied to Hamada light crude oil (42.6° API) transported through the 387-km Hamada–Zawiya pipeline over an operating-temperature range of 10–100°C. The study also proposes the Net Exergy Improvement through Heating (NEI_H) index to assess the balance between pumping-energy savings and thermal-exergy consumption and to identify the operating temperature that maximizes the system's overall energy efficiency [3].

2. Research Problem

The modeling of Hamada crude-oil transportation through the Hamada–Zawiya pipeline lacks an integrated mathematical framework that links the effects of seasonal temperature variations on rheological properties—especially viscosity—to the pipeline's hydraulic response and thermodynamic transport efficiency. This deficiency limits the quantitative assessment of energy losses and the identification of optimal operating conditions.

3. Research Objectives

This research aims to develop an integrated mathematical model that relates temperature to the viscosity of Hamada crude oil using the Andrade model and to analyze the resulting effects on the Reynolds number, friction factor, pressure drop, and pumping power. It also seeks to evaluate transport efficiency through exergy analysis and to propose the NEIH index for identifying the optimal thermal conditions that yield the highest operating efficiency.

4. Significance of the Research:

The significance of this study lies in providing a unified framework that integrates rheological, hydraulic, and thermodynamic analyses for pipeline-performance evaluation. It also introduces the NEIH index as a quantitative tool for assessing the feasibility of crude-oil heating and balancing the reduction in hydraulic losses against the thermal-exergy requirement, thereby contributing to improved crude-oil transport efficiency.

5- Methodology and Data: Study Approach

The study adopted an applied analytical approach that combines laboratory data for Hamada crude oil with mathematical modeling to evaluate the effect of temperature on the hydraulic and thermodynamic performance of the Hamada–Zawiya pipeline. The measured physical and rheological properties of the crude oil were used as model inputs. Temperature was treated as the principal independent variable, while geometric and operating variables were held constant in order to isolate its effect on transport efficiency and exergy indicators. The study relied on primary laboratory data reported in Internal Report No. CO-412, issued by the Zawiya Refinery Laboratory [4], which contains measurements of Hamada crude-oil properties in accordance with the applicable ASTM D standards [5–9]. Table 1 presents the adopted reference properties.

Table 1. Reference Physical Properties of Hamada Crude Oil Used in the Mathematical Model

Property	Value	International Standard	Use in the Mathematical Model
Specific Gravity	0.8127	ASTM D1298	Calculation of Mass Density
API Gravity	42.6	ASTM D1298	Crude-Oil Classification
Pour Point (°C)	-15	ASTM D97	Minimum Limit for Maintaining Continuous Flow
Salt Content (mg/L)	0.65	ASTM D3230	Pipeline-Corrosion Indicator
Water and Sediment	0.02 %	ASTM D1796	Crude-Oil Purity

The transportation-system dataset also included the engineering and operating specifications of the Hamada–Zawiya pipeline, including pipe dimensions, flow rate, internal roughness, reference density, and the coefficient of thermal expansion, as summarized in Table 2.

Table 2. Engineering and Operating Specifications of the Hamada–Zawiya Pipeline

Value	Variable
387 km	Pipeline Length (L)
18 in = 0.4572 m	Internal Diameter (D)
0.2208 m³/s	Volumetric Flow Rate (Q)
120,000 bbl/day	Flow Rate
1.345 m/s	Reference Flow Velocity (V)
0.045 mm	Absolute Roughness (ε)
812.7 kg/m³	Reference Density at 15.6°C
0.00085 °C⁻¹	Coefficient of Thermal Expansion (β)

6. Mathematical Model and Computational Framework:

6.1 Mathematical Model and Governing Equations

The mathematical model is based on an integrated computational framework that links the effect of temperature on the physical and rheological properties of Hamada crude oil to the hydraulic and exergy performance of the Hamada–Zawiya pipeline. The model begins by calculating the temperature-dependent density and viscosity and subsequently relating them to the Reynolds number, friction factor and pressure drop, ultimately enabling the evaluation of exergy destruction using both a conventional indicator and the proposed NEIH index. All calculations were performed using Microsoft Excel and Python.

6.1.1 Density and API Gravity

$$API = (141.5 / SG) - 131.5 \quad (1)$$

$$\rho = SG \times \rho_w \quad (2)$$

$$\rho_T = \rho_0 \left[1 - \beta \left(T - T_0\right)\right] \quad (3)$$

API gravity was calculated from specific gravity using Equation (1), while mass density was obtained from Equation (2). The density was then corrected at different temperatures using the thermal-expansion relation in Equation (3), thereby providing a more realistic representation of the physical properties under operating conditions.

6.1.2 Viscosity: Viscosity is the physical property with the greatest influence on crude-oil transportation. It decreases nonlinearly as temperature increases, with the precise relationship depending on the crude-oil type and composition[10]. Lower viscosity improves flowability and reduces hydraulic losses and energy consumption.

$$\mu = A\, e^{(B/T)} \quad (4)$$

$$\ln(\mu) = \ln(A) + B/T \quad (5)$$

The effect of temperature on dynamic viscosity was represented using the Andrade model, as expressed in Equations (4) and (5). The model constants were estimated from the experimental data, while the kinematic viscosity was calculated from the relation $\nu = \mu/\rho$.

6.1.3 **Hydraulic Variables:**

Under the operating conditions investigated, Hamada crude oil exhibits behavior close to that of a Newtonian fluid. The Reynolds number determines the flow regime through the ratio of inertial to viscous forces. As viscosity decreases with increasing temperature, the Reynolds number rises, thereby affecting the friction factor and the overall hydraulic behavior, which depends jointly on the crude-oil properties, pipeline specifications, and operating conditions.

$$A = \pi D^2 / 4 \ , \qquad V = Q / A \quad (6)$$

$$Re = \rho V D / \mu \quad (7)$$

The pipe cross-sectional area and mean flow velocity were calculated using Equation (6), and the Reynolds number was then obtained from Equation (7) to identify the flow regime used in calculating the friction factor.

6.1.4 Friction Factor:

A higher friction factor increases frictional losses and the pumping power required.
Conversely, the temperature-induced reduction in viscosity increases the Reynolds number, decreases the friction factor and pressure drop, and improves operating efficiency. This effect is particularly significant in the Hamada–Zawiya crude-oil pipeline.

$$f = 0.25 / [log10(\varepsilon/3.7D + 5.74/Re^{0.9})]^2 \qquad (8)$$

The Swamee–Jain equation, Equation (8), was used to calculate the friction factor under turbulent-flow conditions as a function of the Reynolds number and relative roughness ($\varepsilon_r = \varepsilon/D$) [11].

6.1.5 Pressure Drop (ΔP):

$$\Delta P = f\,(L/D)\,(\rho V^2/2) \qquad (9)$$

The pressure drop was calculated using the Darcy–Weisbach equation, Equation (9), which expresses the fundamental relationship among the friction factor, flow velocity, oil density, and pipe dimensions [15].

6.1.6 Exergy-Index Calculations:

Exergy is a measure of energy quality and represents the maximum useful work that can be extracted from a system as it reversibly approaches equilibrium with the surrounding environment. In crude-oil pipelines, friction and pressure losses destroy exergy and increase the pumping-power requirement. Exergy analysis therefore provides a more comprehensive assessment than conventional energy indicators because it evaluates both energy quantity and quality and identifies the actual sources of loss within the system [12].

Because crude-oil transport through the pipeline does not involve a change in chemical composition, this study focuses on physical exergy, which is associated with changes in pressure and temperature. Chemical, kinetic, and potential exergy contributions are comparatively limited and are neglected under the operating conditions considered.

- **Conventional Exergy Index (CEI)**: This index is defined as the ratio of mechanical-exergy destruction at any operating temperature to its reference value at 15.6°C. Because mechanical-exergy destruction is directly proportional to the pressure drop, the conventional index is calculated according to the following equation [19]:

$$CEI = \frac{\Delta P_T}{\Delta P_{15.6}} \qquad (10)$$

Substituting the Darcy–Weisbach pressure-drop relation from Equation (9) yields:

$$CEI = \frac{f(T)\rho(T)}{f_0\ \rho_0} \qquad (11)$$

The CEI has an important limitation: it depends only on variations in friction and density and does not account for the thermal energy required to reach the operating temperature. It may therefore decrease continuously as temperature increases, without indicating whether the heating process itself is economically or thermodynamically justified. This limitation motivated the development of the NEI_H index.

- **Proposed Net Exergy Improvement through Heating Index (NEI_H):** The proposed index was derived from the exergy balance of a steady-state hydraulic system. The rate of mechanical-exergy destruction is expressed as follows:

$$\dot{E}_{dest,mech} = \Delta P\,.\,Q \qquad (12)$$

Substituting the Darcy–Weisbach pressure drop and the flow velocity gives:

$$\dot{E}_{dest,mech} = f\left(\frac{8L\,\rho\,Q^3}{\pi^2 D^5}\right) \qquad (13)$$

To represent the effect of temperature, the friction factor is linked to the Swamee–Jain equation for turbulent flow, while the Andrade model is used to describe the viscosity of Hamada crude oil at any absolute temperature. The thermal exergy required to heat the oil from the reference temperature T0 to the operating temperature T is calculated from the following relation:

$$\dot{E}_{x,th} = \dot{m}\, C_p \left[(T - T_0) - T_0\, ln\, ln\, \left(\frac{T}{T_0}\right)\right] \quad (14)$$

where ṁ is the mass flow rate and Cp is the specific heat capacity of the oil.
Accordingly, the NEI_H index for Hamada crude oil is defined as the ratio of the mechanical-exergy savings produced by heating to the thermal exergy required to achieve those savings, as follows:

$$NEI_H = \frac{\dot{E}_{dest,mech}(T_0) - \dot{E}_{dest,mech}(T)}{\dot{E}_{x,th}(T)} \quad (15)$$

Substitution of the preceding relations yields the final form of the proposed index used in the computational calculations:

$$NEI_H = \frac{\left(\frac{8\, L\, Q^3}{\pi^2\, D^5}\right)\, [\rho_0\, f_0 - \rho(T) f(T)]}{\rho(T)\, Q\, C_P \left[(T - T_0) - T_0 ln\left(\frac{T}{T_0}\right)\right]} \quad (16)$$

where ρ_0 and f_0 are the density and friction factor at the reference temperature, respectively, and $\rho(T)$ and $f(T)$ are the corresponding values at the operating temperature T.

- **Mathematical Derivation of the Optimal Operating Temperature**

To maximize the efficiency of the Hamada crude-oil transportation system on the basis of the NEI_H index, the maximum value of the objective function must be identified by setting its first derivative with respect to the absolute temperature (T) equal to zero, which constitutes the necessary optimality condition:

$$\frac{d(NEI_H)}{dT} = 0 \quad (17)$$

By grouping all temperature-independent parameters into the constant$\left(\wp = \frac{8LQ^2}{\pi^2 D^5 C_P}\right)$, Equation (14) can be expressed in a compact form as the quotient of two temperature-dependent functions, where$[U(T) = \rho_0\, f_0 - \rho(T) f(T)]$ represents the numerator, and $V(T) = \rho(T) \left[(T - T_0) - T_0 ln\left(\frac{T}{T_0}\right)\right]$ represents the denominator

$$NEI_H = \wp\, \frac{U(T)}{V(T)} \quad (18)$$

$$U(T) = -\, \rho(T) \frac{df(T)}{dT} - f(T) \frac{d\rho}{dT} \quad (19)$$

$$V(T) = \rho(T) \left(\frac{T - T_0}{T}\right) + [(T - T_0) - T_0\, ln\, ln\, \frac{T}{T_0}]\ \frac{d\rho}{dT} \quad (20)$$

$$\frac{d(NEI_H)}{dT} = \wp\, \frac{U(T)V(T) - U(T)V(T)}{[V(T)]^2} = 0 \quad (21)$$

Because the relevant coefficients are constant, differentiation yields:
Direct substitution of the functions and their derivatives produces the governing equation for the optimal operating temperature:

$$\left[-\, \rho(T) \frac{df(T)}{dT} - f(T) \frac{d\rho}{dT}\right] \left[\rho(T) \left((T - T_0) - T_0 ln\left(\frac{T}{T_0}\right)\right)\right] = [\rho_0\, f_0 - \rho(T) f(T)] \left[\rho(T) \left(\frac{T - T_0}{T}\right) + \left((T - T_0) - T_0\, ln\, ln\, \frac{T}{T_0}\right) \frac{d\rho}{dT}\right]$$

The governing equation is a highly nonlinear transcendental equation because it couples the implicit temperature derivative of density with the thermal derivative of the friction factor, which is itself governed by the exponential viscosity model for Hamada crude oil. Because the variable cannot be isolated analytically, the coupled system was solved using numerical algorithms.

6.2 Quantitative Analysis Indicators:

To support the interpretation of the results, three quantitative measures were used: absolute difference (AD), percentage change (PC), and sensitivity coefficient (S). These measures quantify the response of the physical, hydraulic, and exergy variables to thermal variation.

6.3 Model Assumptions and Limitations:

The model assumes a constant internal diameter, flow rate, pipe roughness, and steady flow. It neglects the detailed effects of heat transfer, thermal expansion of the pipe, topographic variations, and the deposition of waxes and asphaltenes. Within these assumptions, the model provides an appropriate representation of the hydraulic and exergy performance of the Hamada–Zawiya pipeline.
Future developments may incorporate more detailed heat-transfer models and field operating data.

7-Results and Discussion:

The results were analyzed using an integrated simulation of the hydraulic and exergy performance of the Hamada–Zawiya pipeline. The simulation incorporated the laboratory-measured physical properties of Hamada crude oil together with the adopted engineering and operating specifications of the transportation system. Calculations were conducted over a temperature range of 10–100°C, while all geometric and operating variables were held constant; temperature was therefore the sole independent variable used to assess changes in flow behavior and system efficiency.
The simulation included calculations of the temperature-dependent density and viscosity and an evaluation of the flow characteristics in terms of Reynolds number, friction factor, pressure drop, and hydraulic pumping power. Thermodynamic performance was additionally assessed using both the conventional exergy index and the proposed NEI_H index. Table 3 presents the simulation results, which provide the quantitative basis for analyzing the influence of temperature on the pipeline's hydraulic and exergy performance.

Table 3. Effect of Temperature on the Dynamic, Hydraulic, and Exergy Characteristics of the Hamada Oil Pipeline

Temperature (°C)	Density (kg/m³)	Kinematic Viscosity (cSt)	Dynamic Viscosity μ cP	Reynolds Number (Re)	Friction Factor (f)	Pressure Drop (bar)	Conventional Exergy Index (CEI)	Proposed Exergy Index NEI_H
10	816.3	14.50	11.84	42422	0.0219	137.02	1.0363	-5.3437
15.6	812.7	12.55	10.20	49010	0.0212	132.22	1.0000	-
20	809.8	11.25	9.11	54877	0.0208	128.73	0.9737	6.4777
30	802.8	8.88	7.13	69260	0.0198	121.65	0.9201	1.8942
40	795.8	7.12	5.67	86363	0.0190	115.52	0.8737	1.0743
50	788.9	5.79	4.57	106164	0.0182	110.19	0.8334	0.7350
60	782.0	4.77	3.37	128820	0.0176	105.50	0.7980	0.5510
70	775.1	3.98	3.09	154467	0.0171	101.36	0.7666	0.4365
80	768.2	3.36	2.58	183219	0.0166	97.68	0.7388	0.3588
90	761.3	2.86	2.18	215165	0.0162	94.38	0.7139	0.3029
100	754.4	2.46	1.85	250373	0.0158	91.41	0.6914	0.2611

The experimental and hydraulic results reported in Table 3 reveal a close and dynamic relationship between thermal variation and the physical and operating properties of Hamada crude oil as it flows through the pipeline. This behavior can be analyzed according to the following scientific dimensions:
First: Thermal Variation in Physical Properties (Density and Viscosity):
The results show a uniform, approximately linear decline in mass density as temperature increases,

from 816.3 kg/m³ under winter operating conditions at 10°C to 754.4 kg/m³ at 100°C. This trend is attributable to thermal expansion: as the molecules gain kinetic energy, intermolecular spacing and specific volume increase while mass remains constant, causing the density to decrease progressively. This behavior is expected for Hamada crude oil, which is classified as a structurally stable light crude oil (API gravity = 42.6°), and is consistent with the observations reported by White (2016) [16].
In contrast, the dynamic viscosity decreased exponentially from 11.84 cP at 10°C to 1.85 cP at 100°C, in accordance with the Andrade model. The reduction results from weaker intermolecular attraction among the hydrocarbon molecules and lower resistance to shear as temperature rises. This finding agrees with Dolatyari et al. (2024) [14] and Hamied et al. (2023) [13], who identified viscosity as the crude-oil property most sensitive to thermal variation. The magnitude of the decrease observed in the present study exceeded that reported in some earlier studies, which may be attributed to the high API gravity of Hamada crude oil (42.6°) and its low water-and-sediment content, both of which increase its responsiveness to heating. The results confirm that viscosity is the dominant variable governing hydraulic performance because it drives the subsequent changes in Reynolds number, friction factor, and pressure drop.

Second: Flow Dynamics and Turbulence (Reynolds Number and Friction Factor):
The increase in temperature reduced the dynamic viscosity according to the exponential Andrade relationship, thereby increasing the ratio of inertial forces to viscous forces. Consequently, the Reynolds number increased from 42,422 under winter conditions to 250,373 at 100°C in a nonlinear manner. This behavior agrees with Yuan et al. (2023) [15] and Hamied et al. (2023) [13], who showed that temperature affects the Reynolds number indirectly through viscosity reduction. The increase observed in the present study was more pronounced because of the comparatively low viscosity and high API gravity of Hamada crude oil, which enhanced flowability and maintained turbulent flow throughout the entire operating range.
As the Reynolds number increased, the friction factor decreased markedly from 0.0212 at the reference condition to 0.0158 at 100°C. This decline is consistent with turbulent-flow correlations for relatively rough pipes, such as the Swamee–Jain equation. A higher Reynolds number reduces the thickness of the viscous sublayer adjacent to the wall and diminishes the direct frictional influence of the steel-pipe roughness. This result is consistent with the classical fluid-mechanics principles presented by White (2016) [16] and with the findings of Gong et al. (2024) [17], who demonstrated that pressure variation in pipelines is governed by the combined effects of viscosity and friction factor rather than by either variable alone.

Third: Mechanical Losses and Thermodynamic Analysis (Pressure Drop and Exergy Index):
The continuous decreases in viscosity and friction factor with increasing temperature reduced the pressure drop from 137.02 bar at 10°C to 91.41 bar at 100°C. This behavior is consistent with the Darcy–Weisbach equation, which relates pressure loss directly to flow resistance. The reduction is attributable to lower wall shear stress as viscosity declines, thereby decreasing the mechanical energy dissipated during transportation. The result is consistent with Gong et al. (2024) [17]; however, the decrease observed in the present study was more pronounced because of the low viscosity and high API gravity (42.6°) of Hamada crude oil, which enhanced flowability over the full temperature range investigated.
The NEI_H index displayed a fundamentally different trend from the conventional exergy index. It reached its maximum value within the optimal thermal range and then declined as heating continued. This pattern reflects diminishing thermodynamic returns: beyond the optimum, the additional thermal exergy consumed exceeds the incremental mechanical-exergy savings obtained from reducing frictional losses. The result is consistent with the model proposed by Kılkış (2026) [19], which emphasizes the balance between consumed thermal exergy and recovered mechanical exergy, and it also agrees with Cheng et al. (2018) [18] and the theoretical foundations of exergy analysis presented by Bejan [20] and Moran et al. [21]. These findings confirm that pipeline optimization should seek to

maximize net exergy rather than merely minimize pressure drop. This constitutes the principal scientific contribution of the study, because the NEIH index identifies the optimal operating-temperature range using an integrated thermodynamic criterion that combines transport efficiency with the exergy cost of heating.

Graphical Results:

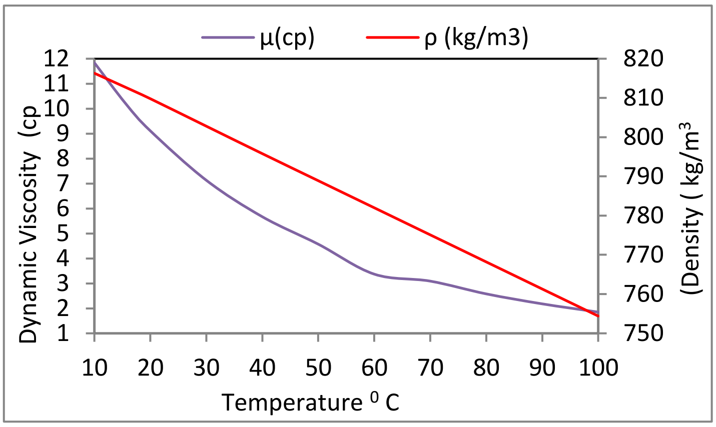

Figure 1. Effect of Temperature on the Density and Dynamic Viscosity of Hamada Crude Oil

Figure 1 presents the coupled behavior of dynamic viscosity (μ) and mass density (ρ) for Hamada crude oil over the temperature range of 10–100°C. The physical trends of the two curves can be interpreted as follows:

The density curve exhibits a gradual and nearly linear decline. This reduction is attributable to the natural thermal expansion of liquids: as the hydrocarbon molecules absorb thermal energy, their molecular motion intensifies and intermolecular spacing increases.

Viscosity exhibits a much stronger exponential sensitivity to temperature, with the steepest decline occurring over the lower-temperature range. This behavior is caused by the increased mobility of hydrocarbon chains and the weakening of intermolecular cohesive forces and physical interactions, which improve flowability inside the pipeline. The observed inverse viscosity–temperature relationship is consistent with the established behavior of hydrocarbon liquids.

The magnitude of viscosity reduction may vary among studies because of differences in crude-oil composition and measurement conditions, particularly the proportions of heavy compounds, resins, and asphaltenes. The thermal response observed for the crude oil investigated therefore reflects its distinct compositional characteristics and confirms the importance of incorporating temperature effects into hydraulic assessments of crude-oil pipelines.

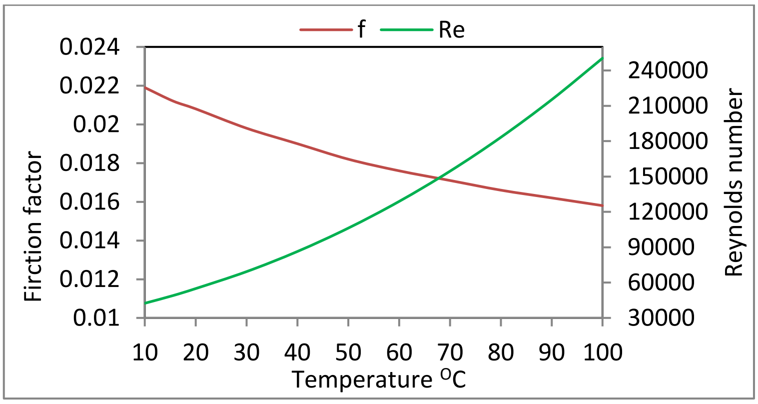

Figure 2. Relationship between Temperature, Reynolds Number, and Friction Factor in the Hamada Oil Pipeline

Figure 2 illustrates the relationship among the investigated variables, and the overall trend reflects the expected physical behavior of the system. The trend arises from the effect of temperature on the rheological and hydraulic properties of crude oil: increasing thermal energy weakens intermolecular attraction and alters the flow characteristics. The result agrees with the general behavior reported in

previous studies, while minor quantitative differences are attributable to variations in crude-oil properties, operating conditions, and the temperature range examined.

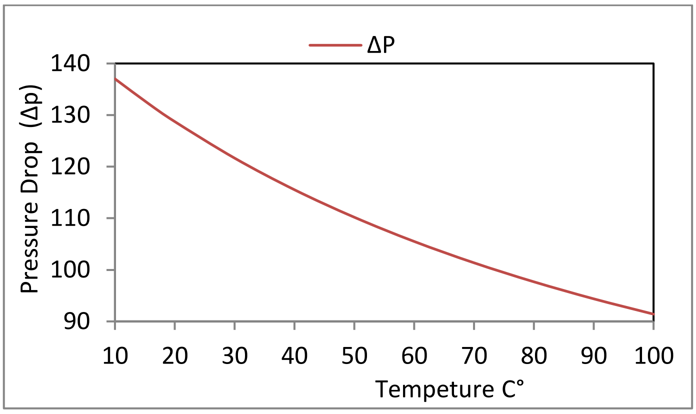

Figure 3. Relationship between Temperature and Hydraulic Performance (Pressure Drop) in the Hamada Oil Pipeline

Figure 3 shows that the pressure drop decreases progressively as temperature increases, falling from 137.02 bar at 10°C to 91.41 bar at 100°C. Heating reduces the oil viscosity, which lowers wall shear stress and the friction factor; consequently, both pressure losses and the energy required to sustain flow decrease. This result is consistent with the broader crude-oil transportation literature, which indicates that higher temperature improves flowability and reduces hydraulic resistance. The trend confirms that operating-temperature control can effectively improve transport efficiency and reduce pumping requirements, particularly in long-distance pipelines.

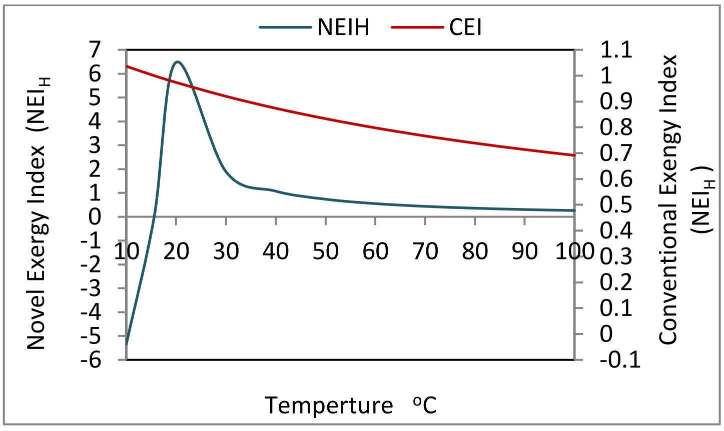

Figure 4. Comparison of the Responses of the Conventional Exergy Index (CEI) and the Proposed NEI_H Index to Thermal Variation

Figure 4 demonstrates a fundamental difference between the responses of the two indices to thermal variation. The CEI decreases almost linearly as temperature increases, reflecting its dependence on frictional and density changes without accounting for the thermal-exergy cost of heating. By contrast, NEIH exhibits a nonlinear response and reaches a distinct maximum of 6.4777 at 20°C. This behavior reflects the interaction between viscosity reduction and hydraulic improvement on the one hand and the increasing thermal-exergy requirement for maintaining the operating temperature on the other. At low temperatures, high viscosity dominates, producing substantial frictional losses and exergy destruction.

Heating the crude oil to 20°C reduces these losses by an amount that exceeds the additional thermal-exergy input, thereby maximizing system efficiency. Beyond this point, the incremental viscosity reduction becomes progressively smaller because of the nonlinear viscosity–temperature relationship, whereas the heating requirement continues to increase; consequently, NEI_H declines. This trend is consistent with previous studies showing that thermally enhanced flow eventually reaches a saturation region in which additional hydraulic gains diminish despite continued energy consumption. The optimum at 20°C is specific to the properties of Hamada crude oil and the operating conditions of the pipeline; other systems may exhibit different optimal temperatures depending on viscosity, flow rate, pipeline dimensions, and operating conditions.

Table 4. Variable and Sensitivity Analysis

Variable	Reference Value, T_0 = 15.6°C	Final Value at 100°C	Absolute Difference	Percentage Change (%)	Sensitivity	Sensitivity Rank
Density	812.7	754.4	58.3	-7.1736	- 0.6907	7
Viscosity	10.20	1.85	8.35	-81.8627	-0.09893	2
Reynolds Number	49010	250373	201363	410.861	2385.82	3
Friction Factor	0.0212	0.0158	0.0054	-25.4716	6.3981	6
Pressure Drop	132.22	91.41	40.81	-30.8652	-0.4835	4
CEI	1.0000	0.69135	0.308652	- 30.865	-0.0036	5
NEI_H	6.4777*	0.2611	6.2166	-95.9692	-.07365	1

(*) Note: The value 6.4777, obtained at 20°C, was used as the baseline for calculating the absolute difference, percentage change, and sensitivity of the proposed NEI_H index. Its value at the reference temperature of 15.6°C is undefined in the mathematical model because the thermal-exergy denominator becomes zero.

Table 4 presents the dynamic sensitivity analysis for Hamada crude oil as the operating system moves from the standard reference condition of 15.6°C to the upper simulation limit of 100°C. Negative sensitivity coefficients predominate, indicating inverse relationships with temperature: as the fluid temperature increases, the corresponding variables decrease quantitatively.

Dynamic viscosity decreased by 81.86%, illustrating the strong thermal sensitivity of the crude oil as intermolecular attraction and resistance to shear weaken with increasing fluid energy. This trend is consistent with crude-oil viscosity-modeling studies that identify viscosity as the principal factor governing temperature-dependent flowability.

The sharp reduction in viscosity directly altered the hydraulic behavior. The Reynolds number increased by 410.86% because the ratio of inertial to viscous forces increased at constant flow rate and pipe diameter. This change reduced the friction factor and pressure drop by 25.47% and 30.86%, respectively. The result is consistent with fundamental pipe-flow principles and with studies showing that hydraulic improvement in crude-oil transportation arises not from viscosity reduction alone, but from the coupled interaction among viscosity, Reynolds number, and wall resistance.

The proposed NEI_H index exhibited the greatest thermal sensitivity among all variables, decreasing by 95.96% from its maximum reference value. This high sensitivity arises because NEI_H does not depend solely on a hydraulic variable; rather, it balances the mechanical-exergy savings obtained from reducing viscous losses against the thermal exergy consumed in raising the oil temperature. Its decline therefore does not indicate deteriorating hydraulic performance. Instead, it identifies the transition from a regime in which heating is beneficial to one in which the incremental benefit of further viscosity reduction becomes smaller than the thermodynamic cost of heat input.

Although mechanical losses continue to decrease as temperature rises, the decline in NEI_H at high temperatures demonstrates the phenomenon of diminishing thermodynamic return. Additional improvements in flowability become limited by the nonlinear viscosity–temperature relationship, whereas thermal-exergy requirements continue to increase. The sensitivity analysis therefore confirms that NEI_H is more comprehensive than the conventional CEI because it captures the balance between hydraulic improvement and the thermodynamic cost of heating.

8- Conclusion

This study developed and evaluated an integrated rheological–thermodynamic model for maximizing crude-oil transportation efficiency through the 387-km Hamada–Zawiya pipeline. By combining turbulent-fluid-flow equations with second-law thermodynamic principles through the proposed NEI_H index and applying sensitivity and optimization analyses, the following principal conclusions were reached:

1. The dynamic sensitivity analysis ranked the proposed NEI_H index first among the variables in terms of responsiveness to thermal variation, with a decrease of 95.96%. It substantially outperformed the conventional CEI, which ranked fifth and exhibited an almost negligible sensitivity coefficient of −0.0036. This finding demonstrates the suitability of NEI_H as a high-resolution objective function for managing thermally influenced pipeline-transport systems.
The numerical solution of the governing optimality condition, $d(NEI_H)/dT = 0$, identified an optimal operating temperature of 20°C, at which the index reached its unique maximum positive value of 6.4777.
2. The narrow temperature range immediately above the reference temperature represents a distinctive physical balance point. Within this range, the rheology of Hamada crude oil responds strongly to heating, producing substantial viscosity reduction and mechanical pumping-energy savings at a comparatively low thermal-exergy cost.
3. The graphical trend and the sharp decline in NEI_H to 0.2611 at 100°C demonstrate that continued heating to high temperatures is neither energetically nor economically justified under the modeled operating conditions. At elevated temperatures, the exergy cost of heat input increases across the long pipeline and outweighs the marginal hydraulic savings achieved at the pumping stations.

9- Recommendations

On the basis of the preceding findings, the following measures are recommended to improve the operational and economic performance of the pipeline:

1. The operators of the Hamada–Zawiya pipeline should maintain the operating temperature close to 20°C to achieve the most favorable exergy balance and minimize overall operating losses.
2. A feasibility study should be conducted for installing crude-oil heating units at the Hamada departure station, particularly for winter operation, in order to raise the crude-oil temperature toward the optimal range, reduce the pressure drop, and improve thermodynamic performance as quantified by NEI_H.
3. The injection of drag-reducing agents during cold winter conditions should be considered as an alternative method for decreasing frictional pressure losses caused by elevated viscosity.
4. Engineers and designers of Libyan oil-transport pipelines are encouraged to adopt NEI_H as a principal criterion for energy-loss assessment rather than relying exclusively on conventional analysis, because of its demonstrated sensitivity to changes in system efficiency.
5. Turbine and pump operation should be adjusted in response to seasonal temperature conditions. In particular, pumping pressure may be reduced automatically during summer to exploit the natural hydraulic advantage associated with lower frictional resistance.

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