



Nonlocal Thermoelastic Analysis of a Porous Solid Subjected to a Moving Internal Heat Source: A Comparative Study Using Coupled, Lord-Shulman, and Green-Lindsay Theories

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Abstract

This study presents a comparative thermoelastic analysis of a nonlocal porous solid subjected to a moving internal heat source. The mathematical model is developed based on three generalized thermoelastic theories: Coupled Dynamics, Lord-Shulman, and Green-Lindsay. Eringen's nonlocal elasticity theory is incorporated to account for small-scale effects. The governing equations of motion, heat conduction, and void distribution are derived and solved analytically using Helmholtz decomposition and normal mode technique. Numerical simulations are performed to examine the influence of the nonlocal parameter, thermal relaxation times, porosity, and heat source velocity on the distributions of displacement, temperature, volume fraction, and stresses. The results demonstrate that nonlocal effects and thermal relaxation times significantly modify the physical fields. The findings provide valuable insights for the design of micro-structured materials and sustainable infrastructure subjected to dynamic thermal loading.

Keywords: Nonlocal elasticity, Porous media, Moving internal heat source, Coupled thermoelasticity, Lord-Shulman theory, Green-Lindsay theory.

الملخص العربي تقدم هذه الدراسة تحليلاً مقارناً للتفاعلات المرنة الحرارية في وسط مسامي لا موضعي ثنائي الأبعاد معرض لمصدر حرارة داخلي متحرك. تم تطوير النموذج الرياضي بالاعتماد على ثلاث نظريات معمة للمرونة الحرارية: النظرية المقترنة، ونظرية لورد-شولمان، ونظرية جرين-ليندسي. تم دمج نظرية المرونة اللاموضعية إيرينجن لحساب تأثيرات المقياس الصغير. تم اشتقاق وحل معادلات الحركة والتوصيل الحراري وتوزيع الفراغات تحليلاً باستخدام تحليل هيلمهولتز وتقنية النمط الطبيعي. أظهرت النتائج العديدة أن المعامل اللاموضعي وأزمة الاسترخاء الحراري والمسامية وسرعة مصدر الحرارة لها تأثير كبير على توزيعات الإزاحة

Nonlocal Thermoelastic Analysis of a Porous Solid Subjected to a Moving Internal Heat Source: A Comparative Study Using Coupled, Lord-Shulman, and Green-Lindsay Theories

ودرجة الحرارة ومجال الكسر الحجمي والإجهادات. النتائج توفر رؤية قيمة لتصميم المواد ذات البنية الدقيقة والبنية التحتية المستدامة المعرضة لأحمال حرارية ديناميكية.

الكلمات المفتاحية: المرونة الغير محلية ، الوسط المسامي، مصدر الحرارة الداخلي المتحرك، النظرية المقترنة، نظرية لورد- شولمان، نظرية جرين- ليندسي.

List of symbols

σ_{ij}	Components of stress tensor
e_{ij}	Components of strain tensor
λ, μ	Lame's constants
α_i	Coefficient of thermal expansion $\beta = (3\lambda + 2\mu)\alpha_i$ such that
δ_{ij}	Kronecker delta, $i, j = x, y$
$\alpha, b, \xi, \omega_0, m, \chi$	Material constants due to the presence of voids
ρ	Density
C_E	Specific heat at constant strain
i	Imaginary, $i = \sqrt{-1}$
ω	Complex time constant
a	Wave number
n_1, n_0	Parameters
τ_0, ν_0	Thermal relaxation times,
K	Thermal conductivity,
T_0	Reference temperature is chosen so that $ (T - T_0)/T_0 \ll 1$,
ϕ	Change in the volume fraction field,
Q	Moving internal heat source
Q_0	Magnitude of an internal heat source
v_0	Velocity of moving internal heat source
$\varepsilon = a_0 e_0$	a_0 an internal characteristic of length and e_0 constant of a material

1. Introduction

Thermoelasticity is an important topic in solid mechanics and has garnered significant attention from scientists and researchers due to its practical implications in engineering and infrastructure. The classical coupled theory of thermoelasticity C-T was introduced based on Fourier's law, which leads to the paradox of infinite thermal wave speed (Chandrasekharaiah, 2023).

To overcome this limitation, Lord and Shulman (1967) proposed a generalized theory with one relaxation time. Later, Green and Lindsay (1972) introduced another generalized theory with two relaxation times. These theories have been widely applied to study thermal shock problems in various materials (Kumar, 2026; Singh, 2024;

Youssef, 2025).

With the development of nanotechnology, Eringen's nonlocal elasticity theory has become essential for studying micro and nano-structured materials (Eringen, 2023; Lim & Yang, 2024). The nonlocal parameter plays a crucial role when the characteristic length is comparable to the internal material length (Ebrahimi et al., 2024).

Porous materials have gained significant attention due to their applications in civil engineering, biomechanics and sustainable construction (Biot, 2024). Several researchers have studied thermoelastic problems in porous media. Sharma (2023) investigated wave propagation, while Othman (2025) analyzed the effect of voids. Abbas (2025) used FEM for moving heat source problems. However, most existing studies considered only a single thermoelastic theory and neglected nonlocal effects. Despite these efforts, there is a lack of comparative studies on nonlocal porous solids subjected to moving internal heat sources using Coupled Dynamics (C-T), Lord-Shulman (L-S) and Green-Lindsay (G-L) theories simultaneously. Therefore, the objective of this paper is to derive the governing equations and compare the three theories for practical applications (Chen & Wang, 2024; Lata, 2024; Abo-Dahab, 2025).

2. Formulation of the problem and basic equation

Consider a homogeneous, isotropic, nonlocal porous solid in the xy-plane subjected to a moving internal heat source. Using Eringen's nonlocal model (Eringen, 2023), the stress-strain-temperature relations are:

$$(1 - \epsilon^2 \nabla^2) \sigma_{ij} = [\lambda (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + b\phi - \beta(1 + \nu_0 \frac{\partial}{\partial t})T] \delta_{ij} + \mu (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}), \quad i, j = 1, 3. \quad (1)$$

where $\nu_0 = 0$ for C-T (chandrasekharaiah, 2023), $\nu_0 = \tau_0$ for L-S (Lord & Shulman, 1967), and $\tau_0 = \nu_0$ for G-L (Green & Lindsay, 1972).

The equations of motion and heat conduction for porous media follow the formulation by Biot (2024) and OTHMAN (2025):

$$\mu \nabla^2 u + (\lambda + \mu) \frac{\partial e}{\partial x} + b \frac{\partial \phi}{\partial x} - \beta(1 + \nu_0 \frac{\partial}{\partial t}) \nabla \frac{\partial T}{\partial x} = (1 - \epsilon^2 \nabla^2) \rho \frac{\partial^2 u}{\partial t^2}, \quad (2)$$

$$\mu \nabla^2 v + (\lambda + \mu) \frac{\partial e}{\partial y} + b \frac{\partial \phi}{\partial y} - \beta(1 + \nu_0 \frac{\partial}{\partial t}) \nabla \frac{\partial T}{\partial y} = (1 - \epsilon^2 \nabla^2) \rho \frac{\partial^2 v}{\partial t^2}, \quad (3)$$

The equation of voids is

$$\alpha \nabla^2 \phi - b e - \xi \phi - \omega_0 b \frac{\partial \phi}{\partial t} + m(1 + \nu_0 \frac{\partial}{\partial t}) T = (1 - \varepsilon^2 \nabla^2) \rho \chi \frac{\partial^2 \phi}{\partial t^2}, \quad (4)$$

The heat conduction equation,

$$K \nabla^2 T = \rho C_E (1 + \tau_0 \frac{\partial}{\partial t}) \dot{T} + \beta T_0 (1 + n_0 \tau_0 \frac{\partial}{\partial t}) \dot{e} + m T_0 (1 + n_0 \tau_0 \frac{\partial}{\partial t}) \dot{\phi} - \rho (1 + n_0 \tau_0 \frac{\partial}{\partial t}) Q. \quad (5)$$

The components of stress tensor are

$$(1 - \varepsilon^2 \nabla^2) \sigma_{xx} = \lambda (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + 2\mu \frac{\partial u}{\partial x} + b\phi - \beta(1 + \nu_0 \frac{\partial}{\partial t}) T, \quad (6)$$

$$(1 - \varepsilon^2 \nabla^2) \sigma_{yy} = \lambda (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + 2\mu \frac{\partial v}{\partial y} + b\phi - \beta(1 + \nu_0 \frac{\partial}{\partial t}) T, \quad (7)$$

$$(1 - \varepsilon^2 \nabla^2) \sigma_{xy} = \lambda (\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}). \quad (8)$$

For simplification, the following non-dimensional variables are used:

$$\begin{aligned} (x', y', \varepsilon') &= \frac{\omega_1^*}{c_0} (x, y, \varepsilon), \quad (u', v') = \frac{\omega_1^*}{c_1} (u, v), \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\mu} \sigma_{ij}, \\ \phi' &= \frac{\omega_1^* \chi_0}{c_1^2} \phi, \quad T' = \frac{T}{T_0}, \quad t' = \omega_1^* t, \quad \Omega' = \frac{\Omega}{\omega_1^*}, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad \omega_1^* = \frac{\rho C_E c_1^2}{k}, \\ \nu'_0 &= \omega_1^* \nu_0, \quad \tau'_0 = \omega_1^* \tau_0, \quad Q' = \frac{c_1^2}{k T_0 \omega_1^{*2}} Q. \end{aligned} \quad (9)$$

From Eq. (9) in to Eqs. (2)-(5) we get

$$\nabla^2 u + E_1 \frac{\partial e}{\partial x} + E_2 \frac{\partial \phi}{\partial x} - E_3 (1 + \nu_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial x} = E_4 (1 - \varepsilon^2 \nabla^2) \frac{\partial^2 u}{\partial t^2}, \quad (10)$$

$$\nabla^2 v + E_1 \frac{\partial e}{\partial y} + E_2 \frac{\partial \phi}{\partial y} - E_3 (1 + \nu_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial y} = E_4 (1 - \varepsilon^2 \nabla^2) \frac{\partial^2 v}{\partial t^2}, \quad (11)$$

$$\nabla^2 \phi - E_5 e - E_6 \phi - E_7 \frac{\partial \phi}{\partial t} + E_8 (1 + \nu_0 \frac{\partial}{\partial t}) T = E_9 (1 - \varepsilon^2 \nabla^2) \frac{\partial^2 \phi}{\partial t^2}, \quad (12)$$

$$\varepsilon_1 \nabla^2 T - E_{10} (\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2}) \phi - \varepsilon_2 (\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2}) e - (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}) \frac{\partial e}{\partial t} = E_{11} (1 + n_0 \tau_0 \frac{\partial}{\partial t}) Q. \quad (13)$$

Also, the constitutive Eqs. (6)-(8) reduces to

$$(1 - \varepsilon^2 \nabla^2) \sigma_{xx} = E_{12} [\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}] + 2\mu \frac{\partial u}{\partial x} + E_2 \phi - E_3 (1 + \nu_0 \frac{\partial}{\partial t}) T, \quad (14)$$

$$(1 - \varepsilon^2 \nabla^2) \sigma_{yy} = E_{12} [\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}] + 2\mu \frac{\partial v}{\partial y} + E_2 \phi - E_3 (1 + \nu_0 \frac{\partial}{\partial t}) T, \quad (15)$$

$$(1 - \varepsilon^2 \nabla^2) \sigma_{xy} = (\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) - \frac{p}{2} (\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}). \quad (16)$$

Where

$$E_1 = \frac{\lambda + \mu}{\mu}, \quad E_2 = \frac{b c_1^2}{\mu \omega_1^{*2} \chi}, \quad E_3 = \frac{\beta T_0}{\mu}, \quad E_4 = \frac{\rho c_1^2}{\mu}, \quad E_5 = \frac{b \chi}{\alpha}, \quad E_6 = \frac{\xi c_1^2}{\alpha \omega_1^{*2}}, \quad E_7 = \frac{\omega_0 c_1^2}{\alpha \omega_1^*},$$

$$E_8 = \frac{mT_0\chi}{\alpha}, E_9 = \frac{\rho c_1^2 \chi}{\alpha}, E_{10} = \frac{mc_1^2}{\rho ce \omega_1^{*2} \chi}, A_{11} = \frac{\omega_1}{c_e}, A_{12} = \frac{\lambda}{\mu}, \varepsilon_1 = \frac{K \omega_1^*}{\rho c_1^2 ce}, \varepsilon_2 = \frac{\beta}{\rho ce}.$$

We define displacement potentials ψ_1 and ψ_2 which relate to displacement components u and v as,

$$u = \frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_2}{\partial y}, \quad \text{and} \quad v = \frac{\partial \psi_1}{\partial y} - \frac{\partial \psi_2}{\partial x}, \quad (17)$$

$$e = \nabla^2 \psi_1, \quad \text{and} \quad \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) = \nabla^2 \psi_2. \quad (18)$$

By substituting from Eq. (18) in Eqs. (10)-(13), this yields

$$[(1 + E_1)\nabla^2 - E_4(1 - \varepsilon^2 \nabla^2)] \frac{\partial^2}{\partial t^2} \psi_1 + E_2 \phi - E_3(1 + \nu_0) \frac{\partial}{\partial t} T = 0, \quad (19)$$

$$[\nabla^2 - E_4(1 - \varepsilon^2 \nabla^2)] \frac{\partial^2}{\partial t^2} \psi_2 = 0, \quad (20)$$

$$-E_5 \nabla^2 \psi_1 + (\nabla^2 - E_6 - E_7 \frac{\partial}{\partial t} - E_9(1 - \varepsilon^2 \nabla^2)) \frac{\partial^2}{\partial t^2} \phi + E_8(1 + \nu_0) \frac{\partial}{\partial t} T = 0, \quad (21)$$

$$-\varepsilon_2 \left(\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2} \right) \nabla^2 \psi_1 - A_{10} \left(\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2} \right) \phi + \varepsilon_1 \nabla^2 T - \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T = E_{11} (1 + n_0 \tau_0) \frac{\partial}{\partial t} \dot{Q}. \quad (22)$$

3 Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$[u, v, T, \psi_1, \psi_2, \phi, \sigma_{ij}](x, y, t) = [\bar{u}, \bar{v}, \bar{T}, \bar{\psi}_1, \bar{\psi}_2, \bar{\phi}, \bar{\sigma}_{ij}](y) \exp[i(\omega t + ax)], \quad (23)$$

$$Q = \bar{Q} \exp(i\omega x - at), \quad \bar{Q} = \bar{Q}_0 y_0 \quad (24)$$

Where $[\bar{u}, \bar{v}, \bar{T}, \bar{\psi}_1, \bar{\psi}_2, \bar{\phi}, \bar{\sigma}_{ij}](y)$ are the amplitudes of the function.

Substituting Eqs. (23) and (24) into governing Eqs. (19)-(22), then we have,

$$(D^2 - S_3) \bar{\psi}_1 + S_4 \bar{\phi} - S_5 \bar{T} = 0, \quad (25)$$

$$(D^2 - S_7) \bar{\psi}_2 = 0, \quad (26)$$

$$-E_5(D^2 - a^2) \bar{\psi}_1 + (S_8 D^2 - S_9) \bar{\phi} + S_{10} \bar{T} = 0, \quad (27)$$

$$S_{11}(D^2 - a^2) \bar{\psi}_1 - S_{12} \bar{\phi} + (D^2 - S_{13}) \bar{T} = E_{11} S_{14} y_0 Q_0. \quad (28)$$

Where,

$$S_1 = 1 + E_1, \quad S_2 = S_1 - E_4 \omega^2 \varepsilon^2, \quad S_3 = \frac{S_2 a^2 - E_4 \omega^2}{S_2} S_4 = \frac{A_2}{S_2}, \quad S_5 = \frac{E_3(1 + i \nu_0 \omega)}{S_2},$$

$$S_6 = (1 - E_9 \omega^2 \varepsilon^2), \quad S_7 = \frac{a^2 S_6 + E_4 \omega^2}{S_6}, \quad S_8 = (1 - E_9 \omega^2 \varepsilon^2),$$

$$S_9 = S_8 a^2 + E_6 - E_7 i \omega - E_9 \omega^2, \quad S_{10} = E_8(1 + i \nu_0 \omega), \quad S_{11} = \frac{-\varepsilon_2(i \omega - n_0 \tau_0 \omega^2)}{\varepsilon_1},$$

$$S_{12} = \frac{E_{10}(i \omega - n_0 \tau_0 \omega^2)}{\varepsilon_1}, \quad S_{13} = \frac{\varepsilon_1 a^2 + i \omega - n_0 \tau_0 \omega^2}{\varepsilon_1}, \quad S_{14} = \frac{1 + n_0 \tau_0 \omega}{\varepsilon_1}.$$

Eliminating $\bar{\psi}_1, \bar{\phi}$ and ψ_1^* between Eqs. (25) - (28), we get

$$[D^6 - A D^4 + B D^2 - C] \{ \bar{\psi}_1(y), \bar{\phi}(y), \bar{T}(y) \} = E_{11} S_{14} Q_0, \quad (29)$$

$$\text{where } A = \frac{1}{S_8} (S_9 - E_5 S_4 + S_3 S_8 + S_8 S_{13} - S_5 S_8 S_{11}),$$

$$B = \frac{1}{S_8} (S_3 S_9 + S_9 S_{13} + S_{10} S_{12} - E_5 S_4 S_{13} - E_5 S_5 S_{12} + S_3 S_8 S_{13} + S_4 S_{10} S_{11} - S_5 S_9 S_{11} - E_5 S_4 a^2 - S_5 S_8 S_{11} a^2),$$

$$C = \frac{1}{S_8} (S_3 S_9 S_{13} + S_3 S_{10} S_{12} - S_5 S_4 S_{13} a^2 - E_5 S_5 S_{12} a^2 + S_4 S_{10} S_{11} a^2 - S_5 S_9 S_{11} a^2).$$

The solution of Eqs. (29) and (26) have the form

$$\bar{\psi}_1(y) = \sum_{n=1}^3 M_n e^{-k_n y} - L_1, \quad (30)$$

$$\bar{\phi}(y) = \sum_{n=1}^3 H_{1n} M_n e^{-k_n y} - L_2, \quad (31)$$

$$\bar{T}(y) = \sum_{n=1}^3 H_{2n} M_n e^{-k_n y} - L_3. \quad (32)$$

$$\bar{\psi}_2(y) = M_4 e^{-r y}, \quad (33)$$

Where, M_n ($n=1,2,3$) are some constants, k_n^2 are the roots of the characteristic equation of Eq. (29).

To obtain the components of the displacement vector, substituting from Eqs. (30) and (43) in Eq. (17), then

$$\bar{u}(y) = \left(\sum_{n=1}^3 i a H_{3n} M_n e^{-k_n y} - r M_4 e^{-r y} \right) - L_4, \quad (34)$$

$$\bar{v}(y) = \left(\sum_{n=1}^3 -k_n H_{4n} M_n e^{-k_n y} - i a r M_4 e^{-r y} \right) - L_5, \quad (35)$$

Also, the constitutive Eqs. (14)-(16) reduces to

$$\bar{\sigma}_{xx}(y) = \sum_{n=1}^3 H_{5n} M_n e^{-k_n y} + L_6, \quad (36)$$

$$\bar{\sigma}_{yy}(y) = \sum_{n=1}^3 H_{6n} M_n e^{-k_n y} + L_7, \quad (37)$$

$$\bar{\sigma}_{xy} = \left(\sum_{n=1}^2 H_{7n} M_n e^{-k_n y} - r M_4 e^{-r y} \right) + L_8. \quad (38)$$

Where,

$$H_{1n} = \frac{[S_{10}(k_n^2 - S_3) + S_4 S_5 (k_n^2 - a^2)]}{(k_n^2 - S_3)(S_8 k_n^2 - S_9) + S_4 E_5 (k_n^2 - a^2)}, \quad H_{2n} = \frac{S_{11}(k_n^2 - a^2) + S_{12} H_{1n}}{(k_n^2 - S_{13})},$$

$$H_{3n} = ia, \quad H_{4n} = -K_n,$$

$$H_{5n} = \frac{[S_{12}(ia H_{3n} - k_n H_{4n}) + 2ia H_{3n} + E_2 H_{1n} - E_3 H_{2n}(1 + i\nu_0 \omega)]}{1 + \varepsilon^2(a^2 - k_n^2)},$$

$$H_{6n} = \frac{[S_{12}(ia H_{3n} - k_n H_{4n}) - 2k_n H_{4n} + E_2 H_{1n} - E_3 H_{2n}(1 + i\nu_0 \omega)]}{1 + \varepsilon^2(a^2 - k_n^2)},$$

$$H_{7n} = ia H_{4n} - k_n H_{3n}, \quad F_0 = \frac{E_{11} S_{14} \nu_0 Q_0}{S_8}, \quad L_1 = \frac{-F_0}{B} (S_8 S_{13} - S_4 S_5),$$

$$L_2 = \frac{-F_0}{B} (S_{11} S_8 - S_5 S_{13}), \quad L_3 = \frac{-F_0}{B} (S_{10} S - S_3 S_{12}), \quad L_4 = i a L_1, \quad L_5 = -k_n L_1$$

$$L_6 = \frac{E_{12}(ia L_4 + E_2 L_2 - E_3 L_3(1 + i\nu_0 \omega))}{1 + \varepsilon^2 a^2}, \quad L_7 = \frac{E_{12}(ia L_4 + E_2 L_2 - E_3 L_3(1 + i\nu_0 \omega))}{1 + \varepsilon^2 a^2},$$

$$L_8 = 0, \quad B = (S_3 S_8 S_{13} + S_4 S_{10} S_{11}) - a^2 (S_3 S_8 + S_8 S_{13} + S_4 S_5) + a^4 S_8, .$$

4 Boundary conditions

To determine the unknown constants appearing the general solution, we impose a set of boundary conditions at the surface $y=0$ of the half -space.

The following conditions are considered:

(i) Thermal condition

The boundary surface is subjected to a prescribed temperature distribution:

$$T(x, 0, t) = f_1 e^{i(\omega t + ax)}, \quad (39)$$

(ii) voids condition

The voids volume fraction is assumed to have zero gradient in the y-direction at the surface:

$$\frac{\partial \phi}{\partial y}(x, 0, t) = 0 \quad (40)$$

(iii) Mechanical condition - Normal traction

A normal mechanical load is applied on the surface:

$$\sigma_{yy}(x, 0, t) = -f_2 e^{i(\omega t + ax)} \quad (41)$$

(iv) Mechanical condition - shear traction

The surface is free from tangential stress, i.e:

$$\sigma_{xy}(x, 0, t) = 0. \quad (42)$$

Here, f_1 and f_2 are constants.

By substituting the general solution into the boundary conditions specified in the previous section, we obtain a system of 4 algebraic equations for the unknown constants M_n ($n=1,2,3,4$). This system can be expressed as:

$$\sum_{n=1}^3 H_{3n} M_n = -f_1 + L_3, \quad \sum_{n=1}^3 H_{1n} M_n = L_2, \quad \sum_{n=1}^3 H_{6n} M_n = -f_2 - L_7, \quad \sum_{n=1}^3 H_{7n} M_n = -L_8, \quad (43)$$

Equating (43) can be written in matrix form as:

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} = \begin{pmatrix} H_{21} & H_{22} & H_{23} & 0 \\ H_{11} & H_{12} & H_{13} & 0 \\ H_{61} & H_{62} & H_{63} & 0 \\ H_{71} & H_{72} & H_{73} & -r \end{pmatrix}^{-1} \begin{pmatrix} f_1 + L_3 \\ L_2 \\ -f_2 - L_7 \\ -L_8 \end{pmatrix}.$$

5 Numerical results and discussion

To examine the influence of different physical parameters, numerical calculations were performed for a copper-like material. The material constants employed in the computations are:

$$\begin{aligned} \lambda &= 7.9 \times 10^{10} \text{ N.m}^{-2}, \quad \mu = 7.78 \times 10^{10} \text{ N.m}^{-2}, \quad K = 386 \text{ W.m}^{-1} \text{K}^{-1}, \\ \alpha_t &= 1.78 \times 10^{-5} \text{ K}^{-1}, \quad \rho = 8954 \text{ kg.m}^{-3}, \quad C_E = 383.1 \text{ J.kg}^{-1} \text{K}^{-1}, \quad T_0 = 293 \text{ K}, \\ \beta &= 2.68 \times 10^6 \text{ N/m}^2 \text{ deg}, \quad \omega_1^* = 3.58 \times 10^{11} / \text{s}, \quad \chi = 1.753 \times 10^{-15} \text{ m}^2, \quad \xi = 1.475 \times 10^{10} \text{ N/m}^2, \\ b &= 1.6 \times 10^{10} \text{ N/m}^{-2}, \quad \alpha = 3.688 \times 10^{-5} \text{ N}, \quad m = 2 \times 10^6 \text{ N/m}^2 \text{ deg}, \quad x = 0.01, \\ \omega_0 &= 0.0787 \times 10^{-3} \text{ N/m}^2 \text{ s}, \quad t = 0.01, \quad \omega = \zeta_0 + i\zeta_1, \quad \zeta_0 = 0.3, \quad \zeta_1 = -0.4, \quad a = 0.9, \\ f_1 &= 1, \quad f_2 = -1, \quad \tau_0 = 0.05 \text{ s}, \quad \nu_0 = 0.5, \quad Q_0 = 0.5 \text{ K}, \quad \nu_0 = 10 \text{ m.s}^{-1}, \quad 0 \leq y \leq 4. \end{aligned}$$

The numerical scheme was applied to evaluate the real parts of the displacement components u and v , temperature T , volume fraction field ϕ , and stress components σ_{xx} , σ_{yy} , σ_{xy} based on the three generalized thermoelastic theories: CT, L-S, and G-L. Two parametric studies were conducted:

Case 1: Effect of Heat Source

Figures 1–6 present the variations for two values of the heat source ($Q_0 = 0.5, 0.1$) at $\varepsilon = 0.1$, $\nu_0 = 10$.

Figure 1 shows that the horizontal displacement u starts with its maximum at the surface and decreases to zero with increasing y . Increasing the heat source from $Q_0 = 0.1$ to $Q_0 = 0.5$ significantly increases the magnitude of u . The G-L theory yields the highest displacement, while the L-S theory gives the lowest values. All cases decay to zero at large distances, satisfying the boundary conditions. Figure 2 illustrates the variation of the vertical displacement v with distance y . The displacement exhibits oscillatory behavior near the surface. For $Q_0 = 0.5$, the amplitude of oscillation is significantly larger

than for $Q_0 = 0.1$, indicating that increasing the heat source enhances the mechanical response. L-S theory shows the most pronounced minimum at $y \approx 0.5$, while G-L theory gives the highest positive peak. All cues damp to zero for $y > 2$, satisfying the far-field boundary condition. This confirms that the disturbance is localized and decays with distance.

Figure 3 shows that the temperature T , attains its maximum value at the surface $y = 0$ and decreases sharply to vanish within $y < 1$. The temperature profile for $Q_0 = 0.5$ lies above that for $Q_0 = 0.1$, demonstrating the direct effect of the heat source magnitude. The three theories, CT, L-S, and G-L, produce nearly identical cues, with very small differences visible only near the surface. The rapid decay to zero indicates the finite speed of thermal wave propagation in generalized thermoelastic theories.

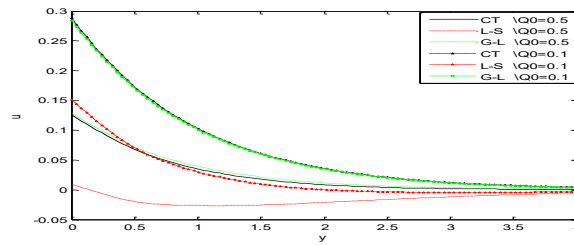


Fig. 1 Distribution of the displacement component u with distance y for two values of heat source at $\varepsilon = 0.1$, $\nu_0 = 10$, $t = 0.01$.

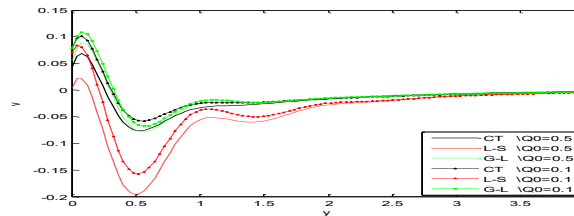


Fig. 2 Distribution of the displacement component v with distance y for two values of heat source at $\varepsilon = 0.1$, $\nu_0 = 10$, $t = 0.01$.

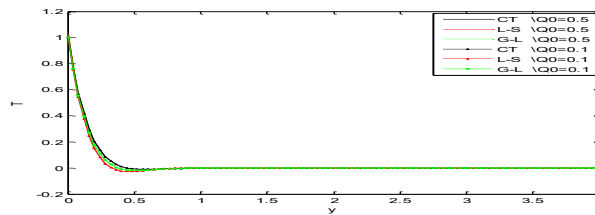


Fig. 3 Distribution of temperature T with distance y for two values of heat source at $\varepsilon = 0.1$, $\nu_0 = 10$, $t = 0.01$.

Figure 4 presents the variation of the volume fraction field ϕ . The field starts with negative values near the surface, reaches a minimum around $y = 0.3 - 0.5$, and then increases to approach zero at large y . Increasing Q_0 from 0.1 to 0.5 leads to a deeper minimum, showing that higher thermal loading causes greater contraction. L-S theory

exhibits the largest magnitude of ϕ , while G-L theory shows the smallest. All curves converge to zero for $y > 1.5$, confirming the satisfaction of boundary conditions.

Figure 5 depicts the normal stress component σ_{yy} . The stress shows a peak near $y = 0.5$ followed by a decay to zero. The magnitude of stress is considerably higher for $Q_0 = 0.5$, compared to $Q_0 = 0.1$. CT theory predicts the highest stress peak, followed by G-L and then L-S theory. The stress becomes negligible for $y > 2$, indicating that the mechanical effect of the heat source is confined to a region close to the surface. The results demonstrate strong coupling between thermal and stress fields. Figure 6 shows the variation of σ_{xx} with distance. The stress starts with a maximum at $y=0$ and decreases monotonically to zero at large distances. For $Q_0 = 0.5$, the stress values are higher than for $Q_0 = 0.1$. Interestingly, for $Q_0 = 0.1$, the three theories almost coincide, while for $Q_0 = 0.5$, CT theory gives the highest values and G-L gives the lowest near the surface. The smooth decay to zero for $y > 3$ confirms the physical consistency of the results.

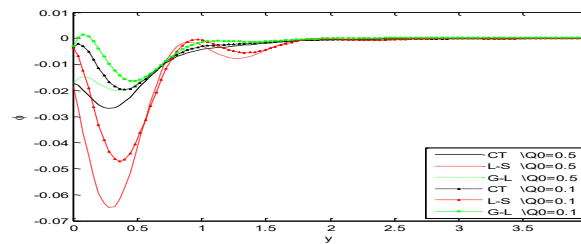


Fig. 4 Distribution of volume fraction ϕ with distance y for two values of heat source at $\varepsilon = 0.1$, $\nu_0 = 10$, $t = 0.01$.

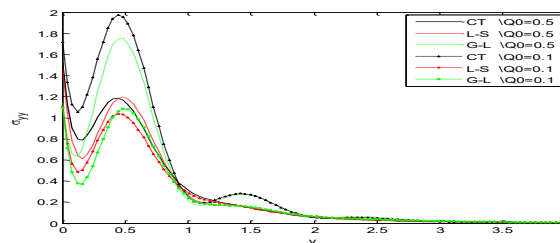


Fig. 5 Distribution of normal stress σ_{yy} with distance y for two values of heat source at $\varepsilon = 0.1$, $\nu_0 = 10$, $t = 0.01$.

Nonlocal Thermoelastic Analysis of a Porous Solid Subjected to a Moving Internal Heat Source: A Comparative Study Using Coupled, Lord-Shulman, and Green-Lindsay Theories

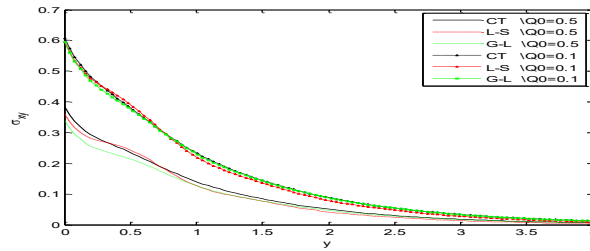


Fig. 6 Distribution of normal stress σ_{yy} with distance y for two values of heat source at $\varepsilon = 0.1$, $\nu_0 = 10$, $t = 0.01$.

General Conclusion for Figs. 1-6

General Conclusion for Figs. 1-6 An increase in the heat source parameter Q_0 leads to an enhancement in all physical quantities. The effect is most visible in displacement and stress components. All fields satisfy the decay condition and vanish at large distances, which validates the numerical solution.

Figure 7 shows that u has a maximum value at ($y = 0$) and decays exponentially to zero with increasing y . For $\varepsilon = 10^{-15}$, the (L-S) theory gives the highest displacement, followed by (C-T) and (G-L). When $\varepsilon = 0.6$, all curves shift downward, indicating that increasing ε reduces nonlocal effects and decreases displacement. The (G-L) theory is the most sensitive to changes in ε . All curves converge for $y > 3$, showing that the effect of ε vanishes far from the source. Figure 8 shows that v starts with a sharp negative value at $y = 0$, then increases to a positive peak around $y = 0.2$ and decreases gradually to zero. For $\varepsilon = 10^{-15}$, the (L-S) theory gives the highest peak, while for $\varepsilon = 0.6$ the three theories are very close. Decreasing ε increases nonlocal effects, leading to larger amplitude of v . For $y > 2.5$ all curves converge, indicating that the influence of ε becomes negligible far from the source. Figure 9 shows that T starts with a maximum value at ($y = 0$) and decreases sharply to reach zero at $y \approx 0.5$. After that, it remains almost constant. The three theories (C-T), (L-S), (G-L) give nearly identical results for both $\varepsilon = 10^{-15}$, and $\varepsilon = 0.6$. This indicates that the nonlocal parameter ε has a minor effect on temperature distribution. The rapid decay reflects the wave nature of heat propagation in generalized thermoelastic theories.

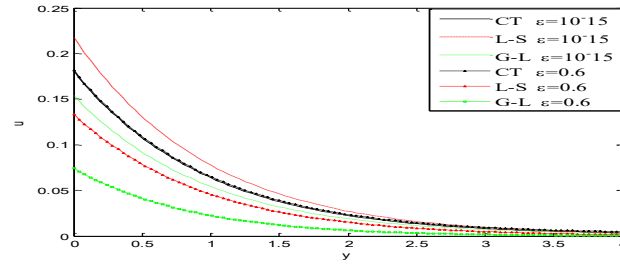


Fig. 7 Distribution of the horizontal displacement component u with distance y for two values of ε at $v_0=10$, $Q_0 = 0.5$, $t = 0.01$.

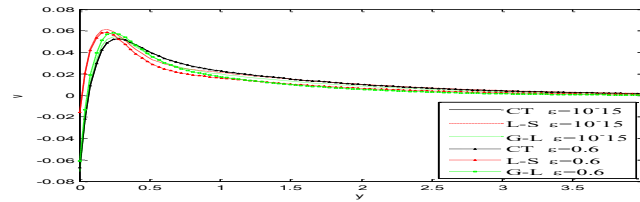


Fig. 8 Distribution of the vertical displacement component v with distance y for two values of ε at $v_0=10$, $Q_0 = 0.5$, $t = 0.01$.

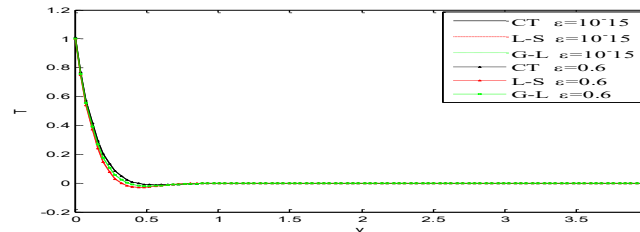


Fig. 9 Distribution of the temperature T with distance y for two values of ε at $v_0=10$, $Q_0 = 0.5$, $t = 0.01$.

Figure 10 shows that ϕ starts with a large negative value at $y = 0$, then increases to a positive peak around $y = 0.3$ before decaying to zero. For $\varepsilon = 10^{-15}$, the (L-S) theory gives the highest peak, while (G-L) gives the lowest. For $\varepsilon = 0.6$, all curves shift downward and the amplitude decreases. This indicates that increasing ε reduces nonlocal effects and leads to a significant reduction in volume fraction. All curves converge to zero for $y > 2$, confirming that the effect of voids diminishes far from the source. Figure 11 shows that σ_{xy} starts with its maximum value at ($y = 0$) and decreases monotonically to approach zero as y increases. For $\varepsilon = 10^{-15}$, the (L-S) theory gives the highest stress, followed by (C-T) and (G-L). For $\varepsilon = 0.6$, all curves shift downward significantly, especially (G-L) which gives the lowest values. This confirms that increasing ε reduces nonlocal effects and leads to lower stress magnitudes. All curves converge at large y , indicating negligible effect of ε far from the source.

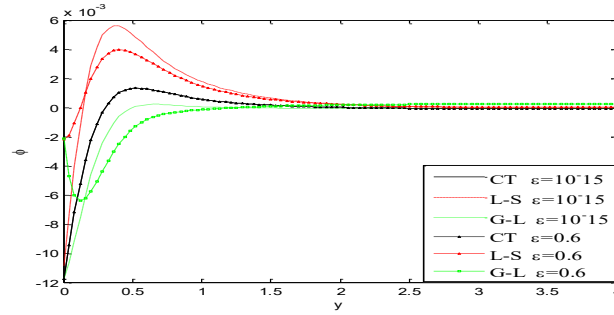


Fig. 10 Distribution of volume fraction ϕ with distance y for two values of ε at $\nu_0=10$, $Q_0 = 0.5$, $t = 0.01$.

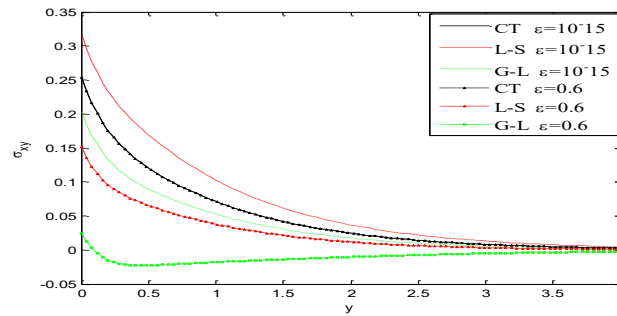


Fig. 11 normal stress σ_{xy} versus y for two values of ε at $\nu_0=10$, $Q_0 = 0.5$, $t = 0.01$.

Figures 12 to 17 illustrate the 3D distributions of displacement, temperature, volume fraction field, and stress components under Green-Lindsay theory. It is clear that all the physical quantities show wave-like behavior along the x direction due to the effect of the moving internal heat source. The values of displacement, temperature, and stresses reach their maximum near the source region and then decrease gradually with increasing x and y until they vanish at large distances. All the functions are continuous and satisfy the physical boundary conditions.

It is also observed that the porosity and the nonlocal parameter have a significant effect on the distribution of all the field variables. The results obtained under Green-Lindsay theory indicate finite speed of thermal wave propagation and higher sensitivity to thermal effects.

Fig. 12 3D variation of the displacement u with the variation of x, y .

Fig. 13 (3D) variation of the displacement component ϕ with the variation of x, y .

Fig. 14(3D) variation of the stress distribution σ_{yy} with the variation of x, y

Fig. 15(3D) variation of the stress distribution σ_{xy} with the variation of x, y .

5. Conclusion

This paper investigates the nonlocal thermoelastic response of a porous solid subjected to a moving internal heat source. The governing equations are formulated and solved analytically using the normal mode technique within the framework of three generalized thermoelastic theories: C-D, L-S, and G-L.

Numerical results for a copper-like material are presented to examine the influence of the moving heat source speed and the nonlocal parameter on the distributions of displacement components, temperature, volume fraction field, and stress components. Based on the obtained results, the following conclusions can be drawn:

1. The moving heat source and the nonlocal parameter have a significant effect on all physical fields. Increasing the heat source speed and the nonlocal parameter leads to an increase in the magnitudes of displacement, temperature, volume fraction, and stresses.
2. The three theories predict different behaviors. The L-S and G-L theories exhibit greater sensitivity to thermal and nonlocal effects than the C-D theory.
3. All computed field variables satisfy the physical condition of vanishing at infinity, confirming the consistency and applicability of the present solution.

In conclusion, the present study demonstrates that nonlocal effects, material porosity, and the speed of the moving heat source must be taken into account for accurate prediction of the thermoelastic behavior of porous materials. The obtained results are of importance in geophysics, structural engineering, and other applications involving porous media subjected to moving thermal loads.

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