



Some General Properties for New Subclasses Defined by a Generalized Differential Operator

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Abstract: in this paper, we define a new generalized multi- parameter differential operator. This operator generalize many well-known operators studied earlier by famous authors. Also, new subclasses of starlike and convex functions will be introduced based on the new operator. Further, some general properties for the new subclasses will be studied.

Key words: analytic functions, differential operator, Hadamard product, starlike function, convex function and growth theorems.

1.Introduction

Assume that given two analytic functions f, g in U where $U = \{z: z \in \mathbb{C}, |z| < 1\}$ and $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$, $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$, then the Hadamard product (or convolution) which is denoted by $(*)$ of these two functions f, g is defined as follows

$$f(z) * g(z) = (f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n,$$

And for several functions $f_1(z), \dots, f_m(z) \in A$,

$$f_1(z) * \dots * f_m(z) = z + \sum_{n=2}^{\infty} (a_{1n} \dots a_{mn}) z^n, \quad z \in U.$$

since A is the class of functions that are analytic in the open unit disk

$U = \{z: z \in \mathbb{C}, |z| < 1\}$ and of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (1)$$

Now, we let

$$\phi_1(z) = \frac{z}{1-z} + \frac{(\lambda_1 + \lambda_2)z}{(1-z)^2} - \frac{(\lambda_1 + \lambda_2)z}{1-z}, \quad \lambda_2 \geq \lambda_1 \geq 0,$$

$$\phi_2(z) = \frac{z}{1-z} + \frac{\lambda_2 z}{(1-z)^2} - \frac{\lambda_2 z}{1-z}, \quad \lambda_2 \geq 0.$$

Then we get the function

$$\begin{aligned} F_m &= \phi_1(z) * \dots * \phi_1(z) \omega_{m\text{-times}} * \frac{z\beta}{(1-z)^2} * \dots * \frac{z\beta}{(1-z)^2} \omega_{\alpha m\text{-times}} \\ &= z + \sum_{n=2}^{\infty} [(\beta n)^\alpha + (\beta n)^\alpha (\lambda_1 + \lambda_2)(n-1)]^m z^n, \end{aligned} \quad (2)$$

Where $m, \alpha \in N_0 = N \cup \{0\}$, $\beta \geq 1$, $\lambda_2 \geq \lambda_1 \geq 0$.

Also, we can get the function

$$\begin{aligned} G_m &= \phi_2(z) * \dots * \phi_2(z) \omega_{m\text{-times}} \\ &= z + \sum_{n=2}^{\infty} [1 + \lambda_2(n-1)]^m z^n, \quad \lambda_2 \geq 0. \end{aligned}$$

And

$$G_m^{-1}(z) = z + \sum_{n=2}^{\infty} \frac{1}{[1 + \lambda_2(n-1)]^m} z^n.$$

Then, by using the convolution we get

$$G_m^{-1}(z) * f(z) = z + \sum_{n=2}^{\infty} \frac{a_n}{[1 + \lambda_2(n-1)]^m} z^n. \quad (3)$$

In 1975, Ruscheweyh [6] defined a differential operator of the k^{th} order of f as follows

$$R^k f(z) = z + \sum_{n=2}^{\infty} c(k, n) a_n z^n, \quad R^k: A \rightarrow A$$

Where $z \in U$, $n \in N_0$, and

$$c(k, n) = \frac{\Gamma(n+k)}{\Gamma(n)\Gamma(k+1)}, \quad k \geq 0.$$

Using the series given by (2), Ruscheweyh differential operator and the convolution given by (3) we define our new generalized differential operator $G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k}$ as follows:

Definition 1.1: for $f \in A$, the new generalized differential operator denoted by

$$G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k}: A \rightarrow A, \quad ,$$

is defined as follows

$$G_{\lambda_1, \lambda_2, \beta}^{m, \alpha} = F_m(z) * R^k f(z) * G_m^{-1}(z) * f(z)$$

If $f(z)$ is given by (1), then we find that

$$G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z) = z + \sum_{n=2}^{\infty} \left[\frac{(\beta n)^{\alpha} + (\beta n)^{\alpha} (\lambda_1 + \lambda_2)(n-1)}{1 + \lambda_2(n-1)} \right]^m c(k, n) a_n z^n. \quad (4)$$

Where $m, \alpha \in N_0 = N \cup \{0\}$, $\beta \geq 1$, $k \geq 0$, $\lambda_2 \geq \lambda_1 \geq 0$.

We assume that

$$M_n = \left[\frac{(\beta n)^{\alpha} + (\beta n)^{\alpha} (\lambda_1 + \lambda_2)(n-1)}{1 + \lambda_2(n-1)} \right]^m$$

So we can write the operator $G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)$ as follows

$$G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z) = z + \sum_{n=2}^{\infty} M_n c(k, n) a_n z^n.$$

Remark. Special cases of this new generalized operator include

i. the Salagean differential operator [7] in the case

$$G_{0,0,1}^{m,1,0} f(z) = S^m f(z) = z + \sum_{n=2}^{\infty} n^m a_n z^n.$$

ii. Al-Oboudi differential operator [2] in the case

$$i. \quad G_{\lambda_1, 0, 1}^{m, 0, 0} f(z) = S_{\lambda}^m f(z) = z + \sum_{n=2}^{\infty} [1 + \lambda(n-1)]^m a_n z^n. \quad \lambda_1 = \lambda.$$

iii . Ruscheweyh differential operator [6] in the case

$$ii. \quad G_{\lambda_1, \lambda_2, \beta}^{0, \alpha, k} f(z) = R^k f(z) = z + \sum_{n=2}^{\infty} c(k, n) a_n z^n.$$

2. Some Relations for $G_{\lambda_1, \lambda_2, \beta}^{m, \alpha}$

Some relations for the differential operator (4) are discussed in the following lemma.

Lemma 2.1. Let $f \in A$, then

$$a) \quad G_{\lambda_1, \lambda_2, \beta}^{0, \alpha, 0} f(z) = f(z)$$

$$b) \quad G_{0,0,1}^{1,1,0} f(z) = z f'(z)$$

Proof:

$$\begin{aligned} a) \quad G_{\lambda_1, \lambda_2, \beta}^{0, \alpha, 0} f(z) &= z + \sum_{n=2}^{\infty} \left[\frac{(\beta n)^{\alpha} + (\beta n)^{\alpha} (\lambda_1 + \lambda_2)(n-1)}{1 + \lambda_2(n-1)} \right]^0 c(0, n) a_n z^n \\ &= z + \sum_{n=2}^{\infty} a_n z^n = f(z). \end{aligned}$$

$$\begin{aligned} b) \quad G_{0,0,1}^{1,1,0} f(z) &= z + \sum_{n=2}^{\infty} \left[\frac{(1n)^1 + (1n)^1 (0+0)(n-1)}{1+0(n-1)} \right]^1 c(0, n) a_n z^n \\ &= z + \sum_{n=2}^{\infty} n a_n z^n = z f'(z). \end{aligned}$$

3. New subclasses.

Starlike functions and convex functions are both classes of analytic functions that have a wide range of applications in complex analysis. These classes of functions are often of interest to many authors and researchers in the field. Recently, many works were done by many authors about these functions and other functions see [1], [3] and [4]. In 1936, Robertson[5] introduced the classes $S^*(\alpha), C(\alpha)$ of starlike and convex functions of order $\alpha, 0 \leq \alpha < 1$, which are defined by

$$S^*(\alpha) = \left[f \in A: \operatorname{Re} \left\{ \frac{z f'(z)}{f(z)} \right\} > \alpha, \quad 0 \leq \alpha < 1, \quad z \in U \right],$$

$$C(\alpha) = \left[f \in A: \operatorname{Re} \left\{ 1 + \frac{z f''(z)}{f'(z)} \right\} > \alpha, \quad 0 \leq \alpha < 1, \quad z \in U \right]$$

by making use of the generalized derivative operator (4) we introduce the new subclasses $S_{m,\alpha}^*(\lambda_1, \lambda_2, \beta, k; \omega)$ and $C_{m,\alpha}(\lambda_1, \lambda_2, \beta, k; \omega)$ of analytic functions as follows:

Definition 3.1. Let $f(z) \in A$, $0 \leq \omega < 1$. Then $f(z) \in S_{m,\alpha}^*(\lambda_1, \lambda_2, \beta, k; \omega)$ if and only if

$$\operatorname{Re} \left\{ \frac{z [G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)]'}{G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)} \right\} > \omega,$$

Where $m, \alpha \in N_0 = N \cup \{0\}$, $\beta \geq 1, k \geq 0, \lambda_2 \geq \lambda_1 \geq 0$.

Definition 3.2. Let $f(z) \in A$, $0 \leq \omega < 1$. Then $f(z) \in C_{m,\alpha}(\lambda_1, \lambda_2, \beta, k; \omega)$ if and only if

$$Re \left\{ \frac{[z, [G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)]']']'}{[G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)]']'} \right\} > \omega,$$

Where $m, \alpha \in N_0 = N \cup \{0\}$, $\beta \geq 1$, $k \geq 0$, $\lambda_2 \geq \lambda_1 \geq 0$.

Now, general properties of these new classes will be studied.

4. General Properties of $S_{m,\alpha}^*(\lambda_1, \lambda_2, \beta, k; \omega)$ and $C_{m,\alpha}(\lambda_1, \lambda_2, \beta, k; \omega)$

In this part, we study the characterization properties and the growth theorems for the function $f(z) \in A$ to belong to the new classes by obtaining the coefficient bounds.

Theorem 4.1. Let $f(z) \in A$. If

$$\sum_{n=2}^{\infty} (n - \omega) M_n c(k, n) |a_n| \leq 1 - \omega, \quad (5)$$

$0 \leq \omega < 1$. Then $f(z) \in S_{m,\alpha}^*(\lambda_1, \lambda_2, \beta, k; \omega)$. The result (5) is sharp.

Proof: assume that the inequality (5) holds. That means

$$\sum_{n=2}^{\infty} (n - \omega) M_n c(k, n) |a_n| \leq 1 - \omega$$

Multiplying both side by 2 we get

$$\sum_{n=2}^{\infty} (2n - 2\omega) M_n c(k, n) |a_n| \leq 2(1 - \omega)$$

And since $(2n - 2\omega) = (n - 1) + (n - 2\omega + 1)$

Then

$$\sum_{n=2}^{\infty} (n - 1) M_n c(k, n) |a_n| + \sum_{n=2}^{\infty} (n - 2\omega + 1) M_n c(k, n) |a_n| \leq 2(1 - \omega),$$

and

$$\sum_{n=2}^{\infty} (n - 1) M_n c(k, n) |a_n| \leq 2(1 - \omega) - \sum_{n=2}^{\infty} (n - 2\omega + 1) M_n c(k, n) |a_n|,$$

where

$$M_n = \left[\frac{(\beta n)^\alpha + (\beta n)^\alpha (\lambda_1 + \lambda_2)(n-1)}{1 + \lambda_2(n-1)} \right]^m$$

using the above inequality along with the triangle inequality $|z| < 1$, we can bound the following complex modulus expression:

$$\begin{aligned} & \left| \frac{z \left(G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z) \right)' - G_{\lambda_1, \lambda_2, \beta}^{m, \alpha} f(z)}{z \left(G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z) \right)' - (2\omega - 1) G_{\lambda_1, \lambda_2, \beta}^{m, \alpha} f(z)} \right| \\ &= \left| \frac{\sum_{n=2}^{\infty} (n-1) M_n c(k, n) a_n z^n}{2(1-\omega)z + \sum_{n=2}^{\infty} (n-2\omega+1) M_n c(k, n) a_n z^n} \right| \\ &\leq \frac{\sum_{n=2}^{\infty} (n-1) M_n c(k, n) |a_n| |z|^{n-1}}{2(1-\omega) - \sum_{n=2}^{\infty} (n-2\omega+1) M_n c(k, n) |a_n| |z|^{n-1}} \\ &\leq \frac{\sum_{n=2}^{\infty} (n-1) M_n c(k, n) |a_n|}{2(1-\omega) - \sum_{n=2}^{\infty} (n-2\omega+1) M_n c(k, n) |a_n|} \leq 1, \end{aligned}$$

then

$$\operatorname{Re} \left\{ \frac{z \left[G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z) \right]'}{G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)} \right\} > \omega$$

Therefore $f(z) \in S_{m, \alpha}^*(\lambda_1, \lambda_2, \beta, k; \omega)$

The proof is complete.

We note that the assertion (5) is sharp and the extremal function is given by

$$f_n(z) = z - \frac{(1-\omega)}{(n-\omega) M_n c(k, n)} z^n, \quad n \geq 2$$

Corollary 4.1. Let the hypotheses of Theorem (4.1) be satisfied. Then

$$|a_n| \leq \frac{(1-\omega)}{(n-\omega) M_n c(k, n)}, \quad \forall n \geq 2$$

Corollary 4.2. Let the hypotheses of Theorem (4.1) be satisfied. Then for

$$m = 1, \quad \omega = \alpha = \lambda_1 = \lambda_2 = k = 0$$

$$|a_n| \leq \frac{1}{n} \quad \forall n \geq 2.$$

Theorem 4.2 : let $f(z) \in A$. If

$$\sum_{n=2}^{\infty} n(n-\omega) M_n c(k, n) |a_n| \leq 1 - \omega, \quad (6)$$

$0 \leq \omega < 1$. Then $f(z) \in C_{m,\alpha}(\lambda_1, \lambda_2, \beta, k; \omega)$. The result (6) is sharp.

Proof. Assume that the inequality (6) holds

$$\sum_{n=2}^{\infty} n(n-\omega) M_n c(k, n) |a_n| \leq 1 - \omega,$$

Multiplying both side by 2 we get

$$\sum_{n=2}^{\infty} 2n(n-\omega) M_n c(k, n) |a_n| \leq 2(1 - \omega).$$

And since $2n(n-\omega) = n(n-1) + n(n-2\omega+1)$

Then

$$\begin{aligned} \sum_{n=2}^{\infty} [n(n-1) + n(n-2\omega+1)] M_n c(k, n) |a_n| &\leq 2(1 - \omega), \\ \sum_{n=2}^{\infty} n(n-1) M_n c(k, n) |a_n| + \sum_{n=2}^{\infty} n(n-2\omega+1) M_n c(k, n) |a_n| \\ &\leq 2(1 - \omega), \end{aligned}$$

By shifting the second summation to the right hand side, we establish the core inequality needed for the absolute bound:

$$\begin{aligned} \sum_{n=2}^{\infty} n(n-1) M_n c(k, n) |a_n| \\ \leq 2(1 - \omega) - \sum_{n=2}^{\infty} n(n-2\omega+1) M_n c(k, n) |a_n|. \end{aligned}$$

To prove that $f(z) \in C_{m,\alpha}(\lambda_1, \lambda_2, \beta, k; \omega)$, we must show that the complex modulus of the structural quotient is bounded by 1 inside the open unit disk $|z| < 1$:

$$\left| \frac{\frac{[z [G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)]']']}{[G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)]']} - 1}{\frac{[z [G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)]']']}{[G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)]']} - (2\omega - 1)} \right| < 1,$$

By finding $[G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)]'$, $z[G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)]'$ and $[z[G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)]']'$, the absolute value become

$$\begin{aligned} & \left| \frac{\sum_{n=2}^{\infty} n(n-1) M_n c(k, n) a_n z^{n-1}}{2(1-\omega) + \sum_{n=2}^{\infty} n(n-2\omega+1) M_n c(k, n) a_n z^{n-1}} \right| \\ & \leq \frac{\sum_{n=2}^{\infty} n(n-1) M_n c(k, n) |a_n|}{2(1-\omega) - \sum_{n=2}^{\infty} n(n-2\omega+1) M_n c(k, n) |a_n|} \leq 1 \end{aligned}$$

Hence

$$\operatorname{Re} \left\{ \frac{[z [G_{\lambda_1, \lambda_2, \beta}^{m, \alpha} f(z)]']']}{[G_{\lambda_1, \lambda_2, \beta}^{m, \alpha} f(z)]']'} \right\} > \omega.$$

Then $f(z) \in C_{m, \alpha}(\lambda_1, \lambda_2, \beta, k; \omega)$. This complete the proof.

The result (6) is sharp and the extremal function is given by

$$f_n(z) = z - \frac{(1-\omega)}{n(n-\omega) M_n c(k, n)} z^n, \quad n \geq 2$$

Corollary 4.3. Let the hypotheses of theorem (4.2) be satisfied. Then

$$|a_n| \leq \frac{(1-\omega)}{n(n-\omega) M_n c(k, n)}$$

In the following, growth theorems are introduced

Theorem 4.3. Let the hypotheses of theorem (4.1) be satisfied. Then for $z \in U$ and $0 \leq \omega < 1$,

$$|z| - \frac{1-\omega}{2-\omega} |z|^2 \leq |G_{\lambda_1, \lambda_2, \beta}^{m, \alpha} f(z)| \leq |z| + \frac{1-\omega}{2-\omega} |z|^2$$

Proof: by using theorem (4.1) we can verify that

$$(2 - \omega) \sum_{n=2}^{\infty} M_n c(k, n) |a_n| \leq \sum_{n=2}^{\infty} (k - \omega) M_n c(k, n) |a_n| \leq 1 - \omega,$$

then

$$\sum_{n=2}^{\infty} M_n c(k, n) |a_n| \leq \frac{(1 - \omega)}{(2 - \omega)}.$$

Now, we can write

$$|G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)| = \left| z + \sum_{n=2}^{\infty} M_n c(k, n) a_n z^n \right|$$

Applying the triangle inequality for $|z| < 1$

$$\begin{aligned} |G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)| &\leq |z| + \sum_{n=2}^{\infty} M_n c(k, n) |a_n| |z|^n \leq |z| + |z|^2 \sum_{n=2}^{\infty} M_n c(k, n) |a_n| \\ &\leq |z| + \frac{(1 - \omega)}{(2 - \omega)} |z|^2. \end{aligned}$$

The other assertion can be proved as follows:

$$\begin{aligned} |G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)| &= \left| z + \sum_{n=2}^{\infty} M_n c(k, n) a_n z^n \right| \geq |z| - \sum_{n=2}^{\infty} M_n c(k, n) |a_n| |z|^n \\ &\geq |z| - |z|^2 \sum_{n=2}^{\infty} M_n c(k, n) |a_n| \\ &\geq |z| - \frac{(1 - \omega)}{(2 - \omega)} |z|^2. \end{aligned}$$

This completes the proof.

In the same way we can get the following result.

Theorem 4.4. let the hypotheses of theorem 4.2. be satisfied. Then for $z \in U$ and

$$0 \leq \omega < 1$$

$$|z| - \frac{(1 - \omega)}{2(2 - \omega)} |z|^2 \leq |G_{\lambda_1, \lambda_2, \beta}^{m, \alpha, k} f(z)| \leq |z| + \frac{(1 - \omega)}{2(2 - \omega)} |z|^2$$

Also, we have the following results

Theorem 4.5. let the hypotheses of theorem 4.1 be satisfied. Then

$$|f(z)| \geq |z| - \frac{(1-\omega)}{(2-\omega) M_2 c(k, 2)} |z|^2$$

And

$$|f(z)| \leq |z| + \frac{(1-\omega)}{(2-\omega) M_2 c(k, 2)} |z|^2$$

Proof: in virtue of theorem 4.1 we have

$$\sum_{n=2}^{\infty} (n-\omega) M_n c(k, n) |a_n| \leq 1-\omega,$$

Since $(2-\omega) \leq (n-\omega)$ and $M_2 c(k, 2) \leq M_n c(k, n)$ for all $n \geq 2$, then

$$(2-\omega) M_2 c(k, 2) \sum_{n=2}^{\infty} |a_n| \leq \sum_{n=2}^{\infty} (n-\omega) M_n c(k, n) |a_n| \leq 1-\omega,$$

also

$$\sum_{n=2}^{\infty} |a_n| \leq \frac{(1-\omega)}{(2-\omega) M_2 c(k, 2)}$$

Thus we obtain

$$\begin{aligned} |f(z)| &= \left| z + \sum_{n=2}^{\infty} a_n z^n \right| \leq |z| + |z|^2 \sum_{n=2}^{\infty} |a_n| \\ &\leq |z| + \frac{(1-\omega)}{(2-\omega) M_2 c(k, 2)} |z|^2. \end{aligned}$$

The other assertion can be proved as follows

$$\begin{aligned} |f(z)| &= \left| z - \sum_{n=2}^{\infty} a_n z^n \right| \geq |z| - |z|^2 \sum_{n=2}^{\infty} |a_n| \\ &\geq |z| - \frac{(1-\omega)}{(2-\omega) M_n c(k, n)} |z|^2 \end{aligned}$$

This completes the proof.

In the same way, we can get the following results.

Theorem 4.6. let the hypotheses of theorem 4.2 be satisfied. Then for $z \in U$, we have

$$|f(z)| \geq |z| - \frac{(1-\omega)}{2(2-\omega) M_2 c(k, 2)} |z|^2,$$

And

$$|f(z)| \leq |z| + \frac{(1-\omega)}{2(2-\omega)M_2c(k,2)}|z|^2$$

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